One-loop Corrections to Scalar and Tensor Perturbations during Inflation in Stochastic Gravity

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Based on the stochastic gravity, we study the loop corrections to the scalar and tensor perturbations during inflation. Since the loop corrections to scalar perturbations suffer infrared (IR) divergence, we consider the IR regularization to obtain the finite value. We find that the loop corrections to the scalar perturbations are amplified by the e-folding; in other words there appear the logarithmic correction, just as discussed by M.Sloth et al. On the other hand, we find that the tensor perturbations do not suffer from infrared divergence.

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I. INTRODUCTION

Inflation provides a natural framework explaining both the large-scale homogeneity of the universe and its small-scale irregularity. Despite its attractive aspects, there are still many unknowns about the inflation theory, since in most models inflation takes place on an energy scale many orders of magnitude higher than that accessible by accelerators. This is why it is necessary to learn all we can about this high energy regime from the signatures left by inflation in the present universe [1–4].

However, when we consider the power spectrum of the curvature perturbation ζ only by linear analysis, many inflation models predict the same results, which are compatible with the observational data, although the fundamental theories are quiet different. To discriminate between different inflationary models, it is important to take into account nonlinear effects [5–16]. In particular, the classical perturbation theory predicts that when we consider most inflation models, the curvature perturbation ζ, which is directly related to the fluctuation of the temperature of CMB, is conserved in the superhorizon region [18–20]. In this case, the primordial perturbation is essentially characterized by the behavior of the background inflaton field near the time of horizon exit. Although this fact makes the computation of the generated primordial perturbations simple, it makes it difficult to discriminate different inflation models. That is why the non-local dependence on the evolution of the background scalar field has been studied among the nonlinear quantum effects such as the loop corrections [9, 10]. Despite their importance, it is difficult to compute these non-linear quantum effects. This is because they contain integrations regarding internal momenta [11–14]. Furthermore, there are several types of nonlinear interactions that induce loop corrections, such as self-interaction of a scalar field and interaction between a matter field and a gravitational field. Depending on the interaction term, we find different loop correction behavior.

Stochastic gravity may be well-suited to computing loop corrections induced from interactions between a scalar field and a gravitational field. Stochastic gravity was proposed as a means of discussing the behavior of the gravitational field on the sub-Planck scale, which is affected by quantum matter fields [21–30]. From our naive expectation, on this energy scale, the quantum fluctuation of the matter field may dominate that of the gravitational field. Based on this insight, Martin and Verdaguer have presented the evolution equation of the gravitational field, which is affected by a quantum scalar field [24]. The effect induced by the quantum matter field is evaluated by the so-called closed time path (CPT) formalism [31–35]. We integrate the action over quantum scalar fields. As a result the evolution equation of the gravitational field is described by a Langevin-type equation, which is called the Einstein-Langevin equation. In general, we need great effort to compute the loop corrections. In stochastic gravity, however, by focusing on non-linear interaction between a scalar field and the gravitational field which gives the leading contribution, we can compute such loop corrections much more easy. Hence, in this paper, using the Einstein-Langevin equations formulated in [24], we evaluate one loop corrections induced by a quantum scalar field.

In our previous work [36], we applied this formalism to the linear perturbations, especially to the curvature perturbation ζ, which is important to consider the imprint on observational data. We find that it reproduces the same results as the prediction obtained by the quantization of the gauge invariant variables [37, 38], except for the limited case. Only when the e-folding from the horizon crossing time to the end of inflation exceeds some critical value, does the Einstein-Langevin equation not give the same result as that of the gauge invariant variables [39]. Hence, we evaluate loop corrections to scalar and tensor perturbations assuming that the e-folding is smaller than the critical value.

In general, loop corrections contain a divergent part. In a quantum field theory in Minkowski spacetime, the divergence usually appears on the high energy scale. To discuss finite and physical quantities, appropriate regularization and renormalization are required. Apart from such an ultraviolet divergence, there may appear another divergence in de Sitter (or quasi de Sitter) spacetime. This infrared problem is important because if we introduce an infrared cut-off to obtain a finite value, which gives a logarithmic correction. Such a logarithmic correction amplifies the perturbations. We also find that there is a crucial difference between the logarithmic correc-

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tions in scalar and tensor perturbations, which is related to the infrared divergence.

In this paper, we consider a minimally coupled single-field inflation as a simple slow-roll inflation model, whose action is given by

$$S[g,\ \phi] \ = \int d^4x\sqrt{-g}\Big[\frac{1}{2\kappa_B^2}(R-2\Lambda_B)+\alpha_B C_{abcd}C^{abcd}+\beta_B R^2-\frac{1}{2}\{g^{ab}\partial_a\phi\partial_b\phi+2V(\phi)\}\Big] \tag{1.1}$$

where $\kappa_B^2 \equiv 8\pi G_B$ is the bare gravitational constant. The subscript "B" represents the values of bare coupling constants. After we regularize divergent parts and renormalize them, we set the renormalized constants as $\alpha=\beta=\Lambda=0$ for simplicity. We also represent the renormalized gravitational constant by $\kappa^2 \equiv 8\pi G$. In order to characterize the slow-roll inflation, we adopt two slow-roll parameters: $\varepsilon \equiv -\dot{H}/H^2$ and $\eta_V \equiv V_{,\phi\phi}/\kappa^2 V$. As for the time variable, we use the conformal time, τ, and represent the time derivative by a prime.

The paper is organized as follows. In Sec. II, we briefly review the basic idea of stochastic gravity and consider the properties of the Einstein-Langevin equation, which describes the evolution of the gravitational field affected by quantum scalar fields. Then we consider perturbations of the Einstein-Langevin equation around an inflationary background spacetime. In Sec. III, we discuss how the loop corrections depend on the potential of the scalar field in diagrammatical language. This part is independent from the computation of the loop corrections in the later sections. In Sec. IV, we consider the perturbation of the Einstein-Langevin equation. Solving this perturbed equation, we can compute the primordial perturbations generated from the quantum fluctuation of the scalar field. The quantum fluctuation of the scalar field is represented by the stochastic variable. In Sec. V, we compute the correlation function of the stochastic variables. Taking into account the results in Sec. IV and Sec. V, we evaluate the loop corrections to the scalar and tensor perturbations. The conclusion and discussion follow in Sec. VII.

II. STOCHASTIC GRAVITY

First we shortly summarize the basic points of stochastic gravity and the Einstein-Langevin equation derived in [24]. It is a generalization of the semi-classical gravity theory. Assuming that quantum fluctuations of matter fields dominate that of the gravitational field, they quantize matter fields but treat gravity as a classical field. The fluctuations of the gravitational field induced through interaction with quantum matter fields are taken into account as stochastic variables. To discuss such gravitational field dynamics, the CTP formalism is useful. They derive the effective equation of motion based on the CTP functional technique applied to a system-environment interaction, more specifically, based on the influence functional formalism of Feynman and Vernon. It is worth while noting that this Langevin-type equation is well-suited not only to understanding the properties of inflation and the origin of large-scale structures in the Universe but also to explaining the transition from quantum fluctuations to classical seeds. In addition to the ordinary Einstein-Hilbert action, this CTP effective action contains two specific terms, which describe the effects induced through interaction with quantum matter fields. One is a memory term, by which the equation of motion depends on the history of the gravitational field itself. The other is a stochastic source ξ_{ab}, which describes quantum fluctuation of a scalar field. The latter is obtained from the imaginary part of the effective action, and as such it cannot be interpreted as a conventional action. Indeed, there appear statistically weighted stochastic noises as a source for the gravitational field. Under the Gaussian approximation, this stochastic variable is characterized by the average value and the two-point correlation function:

$$\langle\xi_{ab}(x)\rangle = 0\ ,$$
$$\langle\xi_{ab}(x_1)\xi_{c'd'}(x_2)\rangle = N_{abc'd'}(x_1,\ x_2)\ , \tag{2.1}$$

where the bi-tensor $N_{abc'd'}(x_1,\ x_2)$ is called a noise kernel, which represents quantum fluctuation of the energy-momentum tensor in a background spacetime, i.e.,

$$\begin{aligned} N_{abc'd'}(x_1,x_2) &\equiv \frac{1}{4}\mathrm{Re}[F_{abc'd'}(x_1,x_2)] \\ &= \frac{1}{8}\langle\{\hat{T}_{ab}(x_1)-\langle\hat{T}_{ab}(x_1)\rangle,\hat{T}_{ab}(x_2)-\langle\hat{T}_{ab}(x_2)\rangle\}\rangle[g]\ , \end{aligned} \tag{2.2}$$

where $\{\hat{X},\hat{Y}\}=\hat{X}\hat{Y}+\hat{Y}\hat{X}$, g is the metric of a background spacetime, and the bi-tensor $F_{abc'd'}(x,y)$ is defined by

$$\begin{aligned} F_{abc'd'}(x_1,x_2) \ \equiv\ & \langle\hat{T}_{ab}(x_1)\hat{T}_{c'd'}(x_2)\rangle[g] \\ & -\langle\hat{T}_{ab}(x_1)\rangle[g]\ \langle\hat{T}_{c'd'}(x_2)\rangle[g]\,. \end{aligned} \tag{2.3}$$

The expectation value for the quantum scalar field is evaluated in the background spacetime g.

Including the above-mentioned stochastic source of ξ_{ab}, the effective equation of motion for the gravitational field is written as

$$G^{ab}[g+\delta g]=\kappa^2\left[\langle\hat{T}^{ab}\rangle_R[g+\delta g]+2\xi^{ab}\right]\ , \tag{2.4}$$

where δg is the metric perturbation induced by quantum fluctuation of matter fields and stochastic source ξ_{ab} is characterized by the average value and the two-point correlation function Eq. (2.1).

Note that this equation is the same as the semiclassical Einstein equation expect for a source term of stochastic variables ξ_{ab}. Furthermore, the expectation value of the energy-momentum tensor includes a nonlocal effect as follows. It consists of three terms as

$$\langle\hat{T}^{ab}\rangle_R[g+\delta g] = \langle\hat{T}^{ab}(x)\rangle[g] + \langle\hat{T}^{(1)ab}[\phi[g],\delta g](x)\rangle[g] \\ -2\int d^4y\sqrt{-g(y)}H^{abcd}[g](x,y)\delta g_{cd}(y) + O(\delta g^2)\,, \tag{2.5}$$

where the expectation value of $\hat{T}^{(1)ab}$ and H^{abcd} are defined below (Eq. (2.6) and (2.8)). The evolution equation for a scalar field depends on the gravitational field. As a result, the expectation value of energy-momentum tensor (Eq. (2.5)) depends not only directly on the spacetime geometry but also indirectly through a scalar field. When we perturb a spacetime as $(g+\delta g)$, two different changes appear in the right hand side of Eq. (2.5). The second term in Eq. (2.5) represents the direct change, which is described by fluctuation of the gravitational field δg as

$$\begin{aligned}&\langle\hat{T}^{(1)ab}[\phi[g],\delta g](x)\rangle \\ &\quad = \Big(\frac{1}{2}g^{ab}\delta g_{cd} - \delta^a_{\ c}g^{be}\delta g_{de} - \delta^b_{\ c}g^{ae}\delta g_{de}\Big)\langle\hat{T}^{cd}\rangle[g], \\ &\quad -\Big\{\Big(1-\frac{2\,\varepsilon}{3}\Big)\rho + \frac{\delta\psi^2}{2a^2}\Big\}\Big(g^{ac}g^{bd} - \frac{1}{2}g^{ab}g^{cd}\Big)\delta g_{cd},\end{aligned} \tag{2.6}$$

where $\delta\psi^2$ is defined in terms of the quantum fluctuation of the scalar field ψ as follows:

$$\delta\psi^2 \equiv \langle\nabla_0\psi\nabla_0\psi + \gamma^{ij}\nabla_i\psi\nabla_j\psi\rangle[g]. \tag{2.7}$$

We can neglect this term safely on the sub-Planck scale because this term is smaller by the order of $(\kappa H)^2$ than the preceding term. To derive this expression, we have used the background evolution equation for a scalar field.

The third integral term in the r.h.s. of Eq. (2.5) represents the effect from the indirect change and is characterized by the dissipation kernel, which is given by

$$H_{abc'd'}(x_1,x_2) = H^{(\mathrm{S})}_{abc'd'}(x_1,x_2) + H^{(\mathrm{A})}_{abc'd'}(x_1,x_2) \tag{2.8}$$

$$H^{(\mathrm{S})}_{abc'd'}(x_1,x_2) = \frac{1}{4}\mathrm{Im}[S_{abc'd'}(x_1,x_2)] \tag{2.9}$$

$$H^{(\mathrm{A})}_{abc'd'}(x_1,x_2) = \frac{1}{4}\mathrm{Im}[F_{abc'd'}(x_1,x_2)]\,, \tag{2.10}$$

where $S_{abc'd'}(x_1,x_2)$ is defined by

$$S_{abc'd'}(x_1,x_2) \equiv \langle T^*\hat{T}_{ab}(x_1)\hat{T}_{c'd'}(x_2)\rangle[g]. \tag{2.11}$$

T^* denotes that we take time ordering before we apply the derivative operators in the energy momentum tensor. As pointed out in [24], only if the background spacetime g satisfies the semiclassical Einstein equation, is the gauge invariance of the Einstein-Langevin equation guaranteed. Hence, in this paper, to guarantee the gauge invariance, we assume the background spacetime satisfies the semiclassical Einstein equation.

The Einstein-Langevin equation Eq. (2.4) contains two different sources. One is a stochastic source ξ_{ab}, whose correlation function is given by the noise kernel. From the explicit form of a noise kernel Eq. (2.2), we find that ξ_{ab} represents the quantum fluctuation of the energy momentum tensor. The other is an expectation value of the energy momentum tensor in the perturbed spacetime $(g+\delta g)$, which includes a memory term. The integrand of a memory term consists of a dissipation kernel and fluctuation of the gravitational field. To investigate the evolution for fluctuation of the gravitational field, it is necessary to calculate the quantum correction of a scalar field and evaluate the noise kernel and the dissipation kernel. Note that the noise kernel and the dissipation kernel correspond to the contributions from internal lines or loops of the Feynman diagrams, which consist of propagators of a scalar field and do not include those of the gravitational field in our approach.

III. GENERIC FEATURE FROM THE VERTEX OPERATORS

Before we discuss loop corrections to the primordial perturbations in detail, we consider the dependence of loop corrections on a potential of a scalar field diagrammatically.

First we divide a scalar field ϕ into the classical part ϕ_{cl} and the part of small quantum fluctuation ψ. Inflation is mainly driven by ϕ_{cl}. Expanding a potential of the scalar field $V(\phi)$ around ϕ_{cl}, we can write

$$\begin{aligned}V(\phi) &= V(\phi_{cl})\Big[1 + \frac{V'}{V}(\phi_{cl})\,\psi + \frac{1}{2!}\frac{V''}{V}(\phi_{cl})\,\psi^2 + \cdots\Big] \\ &= V(\phi_{cl})\sum_{m=0}^{\infty}\frac{1}{m!}\alpha^{(m)}\,(\kappa\psi)^m,\end{aligned} \tag{3.1}$$

where the coefficient $\alpha^{(m)}$ is defined by

$$\alpha^{(m)} \equiv \frac{d^mV/d\phi^m(\phi_{cl})}{\kappa^m V(\phi_{cl})}\,. \tag{3.2}$$

Taking into account that during inflation ϕ_{cl} changes on the Planck scale, we have normalized $\alpha^{(m)}$ by the Planck mass.

Similarly, we also perturb the gravitational field in the total action. Expanding the total action Eq. (1.1) in terms of the fluctuation of the gravitational field δg and the fluctuation of the scalar field ψ, we find that the following vertices appear:

$$\alpha^{(1)}\delta g\,\psi\,,\;\; \alpha^{(2)}\delta g\,\psi^2\,,\;\; \alpha^{(3)}\delta g\,\psi^3\,,\;\; \alpha^{(4)}\delta g\,\psi^4\;\cdots. \tag{3.3}$$

Although the kinetic term also includes fluctuation terms such as $\delta g\psi$ and $\delta g\psi^2$, since in this section we are interested in information about $V(\phi)$, which is obtained through the coefficients $\alpha^{(m)}$, we do not pay attention to these terms. Of course, when we evaluate the loop corrections in the later sections, we take into account all the fluctuation terms.

The vertex operators given by Eq. (3.3) correspond to the vertex diagrams depicted in Fig. 1. The solid line represents

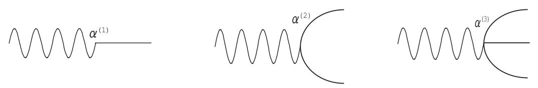

FIG. 1: Vertices

the propagation of the scalar field ψ. The equation of motion for the field in the interaction picture is discussed in Sec. V A. As we will see later, the propagator in the inflationary universe is proportional to H^2. Hence, each solid line contributes as $(\kappa H)^2$. On the other hand, the wavy line represents the propagation of the gravitational field δg. The coefficient $\alpha^{(m)}$ is a coupling constant of the interaction described by the vertex of $\delta g\,\psi^m$. Since the higher loop graphs are suppressed further by $(\kappa H)^2$, we can discuss the quantum corrections by an iterative perturbation method.

To consider the evolution equation of the gravitational field, we integrate out only the degree of freedom of a scalar field. This means that when we evaluate the effective action in the CTP formalism, the gravitational field is treated as a classical external field. Hence the gravitational field contributes as the external line but not as the internal line, in the effective action. We represent the gravitational field by the wavy line. Taking into account this fact, we find the leading contribution to the effective action is given by the diagram depicted in Fig. 2 (1). The amplitude of this leading diagram is proportional

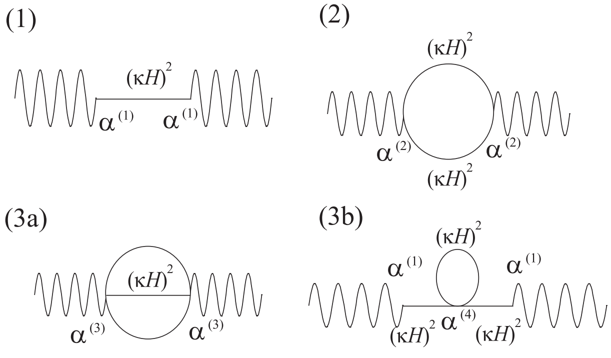

FIG. 2: Feynman diagrams.

to $(\alpha^{(1)}\kappa H)^2$; it depends both on the Hubble parameter H and on the first order derivative of the potential, $\alpha^{(1)}$. The contribution from this tree-level graph corresponds to quantum corrections in the linear perturbation analysis, and it has been given in our previous work [36]. In the next leading order depicted by Fig. 2 (2), the amplitude is proportional to $(\alpha^{(2)})^2(\kappa H)^4$. It depends on the Hubble parameter H and the second order derivative of the potential, $\alpha^{(2)}$. In this paper, we evaluate this leading contribution among loop corrections. We just give a few comments on further order contributions. As seen from Fig. 2 (3), there are two different diagrams, whose amplitudes are proportional to $(\kappa H)^6$. Figure 2 (3a) depends on the third-order derivative of the potential, $\alpha^{(3)}$, while Fig.2 (3b) depends on the fourth-order derivative of the potential, $\alpha^{(4)}$. Figure 2 (3b) comes from the self-interaction of the scalar field. On the other hand, the other three graphs (Fig. 2 (1), (2) and (3a)) are due to the interaction between the gravitational field and the scalar field ($\delta g\,\psi^m (m=1,2,3)$). From our discussion here, the higher-order loop corrections, although they are suppressed by the Planck scale, make it possible to know the more information about the potential of the scalar field. If we can detect these loop corrections, it helps to discriminate many inflation models, even if they cannot be distinguished only from the linear perturbation analysis.

IV. PERTURBATION OF EINSTEIN-LANGEVIN EQUATION

Next we discuss the behavior of the loop corrections, especially those in the superhorizon region. We consider the time evolution of the leading loop corrections, which is depicted in Fig. 2 (2), in the superhorizon region. To calculate the loop corrections to the scalar perturbations and the tensor perturbations, we adopt the following metric form:

$$
\begin{aligned}
ds^2 &= -a^2(\tau)(1+2\mathcal{A}_{\boldsymbol{k}}Y_{\boldsymbol{k}})d\tau^2 - 2a^2(\tau)\frac{k}{\mathcal{H}}\Phi_{\boldsymbol{k}}Y_{j\boldsymbol{k}}d\tau dx^j \\
&\quad + a^2(\tau)\,(\gamma_{ij}+2H^{(t)}_{T\boldsymbol{k}}e_{ij}(\boldsymbol{k})Y_{\boldsymbol{k}})dx^i dx^j\,, \qquad (4.1)
\end{aligned}
$$

where a and γ_{ij} are the scale factor and the metric of maximally symmetric three space. The scalar perturbations are described by $\mathcal{A}$ and $(k/\mathcal{H})\Phi$, which are the so-called lapse function and shift vector, respectively. $H^{(t)}_T$ is the tensor perturbation. The scalar perturbations are expanded by a complete set of harmonic function $Y_{\boldsymbol{k}}(\boldsymbol{x})$, which satisfies

$$
(\Delta + k^2)Y_{\boldsymbol{k}}(\boldsymbol{x}) = 0\,. \qquad (4.2)
$$

Using these harmonic functions, we find the scalar components of vector variables are expanded by

$$
Y_{j\boldsymbol{k}} \equiv -k^{-1}Y_{\boldsymbol{k}|j}\,. \qquad (4.3)
$$

The tensor perturbations are expanded by the basis $e_{ij}(\mathbf{k})$, which satisfies the transverse traceless condition, $\gamma^{ij}e_{ij}(\mathbf{k})=0$ and $k^i e_{ij}(\mathbf{k})=0$. Hence our variables are now $\mathcal{A}_{\boldsymbol{k}}$, $\Phi_{\boldsymbol{k}}$ and $H^{(t)}_{T\boldsymbol{k}}$.

Since this gauge choice fixes both the time slicing and the spatial coordinate completely, all physical variables with this ansatz are gauge invariant. This choice of the time coordinate is called a flat slicing, because the spatial curvature vanishes in this slicing.

Note that because of nonlinear perturbations, the tensor perturbations are not decoupled from the scalar perturbations, and the quantum fluctuations of the scalar field may amplify not only the scalar perturbations but also the tensor perturbations.

A. Scalar perturbations

First we consider loop corrections to the scalar perturbations. In particular, we discuss the evolution of the gauge-invariant variable ζ. We focus on proper nonlinear effects, and then we neglect the contributions from the product of the linear perturbations. Then ζ is related to the density perturbation in a flat slicing ($\delta_f \equiv \delta\rho/\rho$) as

$$\zeta = \frac{1}{2\varepsilon}\delta_f \,. \tag{4.4}$$

This variable ζ is gauge-invariant and turns out to be a curvature perturbation in a uniform density slicing. In the classical perturbation theory, the energy conservation law implies that this variable is conserved in a superhorizon region for a single-field inflation [18–20]. ζ is directly related to a gravitational potential at the late stage of the universe and then to the observed CMB fluctuations. Hereafter, when we need not clarify the mode $\boldsymbol{k}$, we neglect the index of the momentum for the perturbed variables.

The density perturbation in the present slicing is given by

$$\begin{aligned}\delta T^0_{\ 0} &\equiv -\rho\delta_f Y \\ &= \delta g_{0c}\langle \hat{T}^{0c}(x)\rangle[g] + g_{0c}\{\langle \hat{T}^{(1)0c}[\phi[g],\delta g](x)\rangle \\ &\quad -2\int d^4y\sqrt{-g(y)}H^{0cde}[g](x,y)\delta g_{de}(y) + 2\xi^{0c}\}\,.\end{aligned} \tag{4.5}$$

Since the background energy-momentum tensor is given by

$$\langle \hat{T}^{0a}(x)\rangle[g] = g^{0b}\langle \hat{T}^a_{\ b}(x)\rangle[g] = -\rho\, g^{0a}\,, \tag{4.6}$$

the direct contribution from the gravitational field is described as

$$\langle \hat{T}^{(1)00}[\phi[g],\delta g](x)\rangle = -\frac{2}{a^2}\Big\{1+\frac{\varepsilon}{3}+O((\kappa H)^2)\Big\}\rho\mathcal{A}Y\,. \tag{4.7}$$

With these two relations (4.5) and (4.7), the density perturbation in flat slicing is written as

$$\delta_f \simeq -\frac{2\varepsilon}{3}\mathcal{A} + \frac{2}{\rho}(\delta\rho_m + \delta\rho_\xi)\,, \tag{4.8}$$

where we have defined the density perturbations of the stochastic source $\xi^a_{\ b}$ and of the memory term as follows:

$$\begin{aligned}\delta\rho_m &\equiv \int d^3\boldsymbol{x} e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\Big[g^{00}\int d^4y\sqrt{-g(y)} \\ &\qquad \times H_{00c'd'}[g](x,y)g^{c'e'}g^{d'f'}\delta g_{e'f'}(y)\Big]\,, \\ \delta\rho_\xi &\equiv \int d^3\boldsymbol{x} e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\Big[-g_{00}\,\xi^{00}(x)\Big]\,.\end{aligned} \tag{4.9}$$

The Hamiltonian constraint equation gives a relation between the gauge invariant variable $\mathcal{A}$ and the density perturbation δ_f as

$$\mathcal{A} = \frac{1}{3}\Big(\frac{k}{\mathcal{H}}\Big)^2\Phi - \frac{\delta_f}{2}\,. \tag{4.10}$$

Using it, we eliminate $\mathcal{A}$ in Eq. (4.8), and find

$$\Big(1-\frac{\varepsilon}{3}\Big)\delta_f = \frac{2}{\rho}(\delta\rho_\xi + \delta\rho_m) + O\Big((k/\mathcal{H})^2\Big)\,. \tag{4.11}$$

Hence, in superhorizon region, the two-point correlation function for δ_f is expressed in terms of four correlation functions of $\delta\rho_\xi$ and $\delta\rho_m$, i.e.,

$$\begin{aligned}\langle \delta_{f\boldsymbol{k}}(\tau)\delta_{f\boldsymbol{p}}(\tau)\rangle \simeq \frac{4}{V(\tau)^2}\Big[&\langle \delta\rho_{\xi\boldsymbol{k}}(\tau)\delta\rho_{\xi\boldsymbol{p}}(\tau)\rangle \\ &+\langle \delta\rho_{m\boldsymbol{k}}(\tau)\delta\rho_{\xi\boldsymbol{p}}(\tau)\rangle + \langle \delta\rho_{\xi\boldsymbol{k}}(\tau)\delta\rho_{m\boldsymbol{p}}(\tau)\rangle \\ &+\langle \delta\rho_{m\boldsymbol{k}}(\tau)\delta\rho_{m\boldsymbol{p}}(\tau)\rangle\Big]\,.\end{aligned} \tag{4.12}$$

Here we have used the relation

$$V(\tau) = \Big(1-\frac{\varepsilon}{3}\Big)\rho + O(\rho\,(H/m_{pl})^2)\,. \tag{4.13}$$

B. Tensor perturbations

In a similar way to the scalar perturbations, the transverse traceless part of the fluctuation of the energy-momentum tensor is given by

$$\Big[\delta T^i_{\ j}\Big]_{TT} = \Big\{-\frac{\delta\psi^2}{a^2}\,H^{(t)}_T + 2\Big(p\pi^{(t)}_m + p\pi^{(t)}_\xi\Big)\Big\}e^i_{\ j}(\boldsymbol{k})Y_{\boldsymbol{k}}\,, \tag{4.14}$$

where we have defined the transverse traceless part of the anisotropic pressure both for the memory term and the stochastic variable $\xi^a_{\ b}$ as

$$\begin{aligned}p\pi^{(t)}_m e^i_{\ j}(\boldsymbol{k}) &\equiv \int d^3\boldsymbol{x} e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\Big[-g^{ik}\int d^4y\sqrt{-g(y)} \\ &\qquad \times H_{kjc'd'}[g](x,y)g^{c'e'}g^{d'f'}\delta g_{e'f'}(y)\Big]_{TT}\,, \\ p\pi^{(t)}_\xi e^i_{\ j}(\boldsymbol{k}) &\equiv \int d^3\boldsymbol{x} e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\,[g_{jk}\xi^{ik}(x)]_{TT}\,.\end{aligned} \tag{4.15}$$

Taking into account that the transverse traceless part of Einstein tensor is written as

$$\Big[\delta G^i_{\ j}\Big]_{TT} = \frac{1}{a^2}[\,\partial^2_\tau + 2\mathcal{H}\partial_\tau + k^2\,]H^{(t)}_T e^i_{\ j}(\boldsymbol{k})Y_{\boldsymbol{k}}\,, \tag{4.16}$$

we find the transverse traceless part of the Einstein-Langevin equation as

$$\begin{aligned}&(\partial^2_\tau + 2\mathcal{H}\partial_\tau + k^2)H^{(t)}_{T\boldsymbol{k}}(\tau) \\ &\quad = 2a^2\kappa^2\Big(p\pi^{(t)}_{m\boldsymbol{k}}(\tau) + p\pi^{(t)}_{\xi\boldsymbol{k}}(\tau)\Big) \\ &\quad \equiv J_{t\boldsymbol{k}}(\tau)\,.\end{aligned} \tag{4.17}$$

The l.h.s. of this equation is the same as the evolution equation for linear perturbation. In contrast to the linear perturbation analysis, where the tensor perturbations are decoupled from the scalar perturbations, the non-linear interaction couples these two modes. That is why in the r.h.s. there appears

the influence of quantum fluctuations of a scalar field. A linear second-order differential equation with a source term is solved by the retarded Green function constructed from two independent general solutions. In the present case, since we are interested in the tensor perturbations amplified by quantum scalar fields, we assume that the tensor perturbations were absent in the beginning of inflation. This gives $H^{(t)}_{T\boldsymbol{k}}(\tau_i) = 0$ as the initial condition. The two independent general solutions for Eq. (4.17) are given by

$$h_k^{(1)}(\tau) = \frac{x^{\frac{1}{2}}}{a(\tau)} H_\nu{}^{(1)}(x)\,, \quad h_k^{(2)}(\tau) = \frac{x^{\frac{1}{2}}}{a(\tau)} H_\nu{}^{(2)}(x)\,, \tag{4.18}$$

where $\nu^2 = \frac{9}{4} + 3\varepsilon$ and $x = -k\tau$. Hence, we find the solution for Eq.(4.17) as

$$H^{(t)}_{T\boldsymbol{k}}(\tau) = \int_{\tau_i}^{\infty} d\tau' G_{\mathrm{ret\ k}}(\tau,\tau') J_{t\ \boldsymbol{k}}(\tau')\,, \tag{4.19}$$

where the retarded Green function is given by

$$G_{\mathrm{retk}}(\tau,\tau') = \frac{h_k^{(1)}(\tau) h_k^{(2)}(\tau') - h_k^{(2)}(\tau) h_k^{(1)}(\tau')}{W_k(\tau')} \theta(\tau - \tau') \tag{4.20}$$

with

$$W_k(\tau) = h_k^{(2)}(\tau)\, \frac{d}{d\tau} h_k^{(1)}(\tau) - h_k^{(1)}(\tau)\, \frac{d}{d\tau} h_k^{(2)}(\tau)\,. \tag{4.21}$$

Substituting general solutions into these equations, we obtain the corresponding retarded Green function as

$$\begin{aligned} &G_{\mathrm{ret\ k}}(\tau,\tau') \\ &\quad = -\frac{\pi}{2} \frac{a(\tau')}{a(\tau)} \sqrt{\tau\tau'}\, \mathrm{Im}[H_\nu^{(1)}(x) H_{\nu'}^{(2)}(x')]\, \theta(\tau - \tau')\,. \end{aligned} \tag{4.22}$$

Here we have used the formula for the Hankel functions:

$$H_\nu^{(1)}(x) \frac{d}{dx} H_\nu^{(2)}(x) - H_\nu^{(2)}(x) \frac{d}{dx} H_\nu^{(1)}(x) = \frac{4}{\pi i x}\,. \tag{4.23}$$

Substituting these expressions into Eq. (4.19), we find the tensor perturbations amplified by quantum scalar field as

$$\begin{aligned} &H^{(t)}_{T\boldsymbol{k}}(\tau) \\ &= -\frac{\pi}{2} \int_{\tau_i}^{\tau} d\tau' \frac{a(\tau')}{a(\tau)} \sqrt{\tau\tau'}\, \mathrm{Im}[H_\nu^{(1)}(x) H_\nu^{(2)}(x')]\ J_{t\boldsymbol{k}}(\tau') \\ &= -\frac{\pi}{2k^2} \int_x^1 dx' \left(\frac{x}{x'}\right)^{1+\varepsilon} \sqrt{xx'}\, \mathrm{Im}[H_\nu^{(1)}(x) H_\nu^{(2)}(x')] J_{t\boldsymbol{k}}(x')\,. \end{aligned} \tag{4.24}$$

Here we have used the fact that the scale factor scales as $a(\tau) \propto |\tau|^{-(1+\varepsilon)}$. We have also omitted the contribution from the subhorizon region because the Hankel functions oscillate where x is larger than one.

V. NOISE KERNEL

The scalar perturbation ζ and the tensor perturbation $H_T^{(t)}$ are given by the stochastic variable and the memory term. To evaluate the correlation functions for ζ and $H_T^{(t)}$, it is necessary to compute quantum corrections for the scalar field, which are imprinted on the noise and dissipation kernels. It is expected that the contribution from the memory terms $\delta\rho_m$ is smaller than that from the stochastic variable $\delta\rho_\xi$ by the order of magnitude of the slow-roll parameters. The reason is as follows. The dissipation kernel is defined as two-point function of the energy-momentum tensor. As is seen from the definition of $\alpha^{(m)}$, the contribution from the potential term is suppressed by the slow-roll parameters. Also, as summarized in Appendix B of our paper [36], the Green function scales as $(-\tau)^{|\text{slow-roll parameter}|}$ in the superhorizon region. Then, the time derivative of this Green function is suppressed by the slow-roll parameters. Taking into account that only the contribution in the superhorizon region can accumulate on the time integral of the memory term, we can see that the contribution from the memory term, which is proportional to the dissipation kernel, is suppressed by the slow-roll parameters. Neglecting the contribution from the memory term, we find the density perturbation δ_f only in terms of the density perturbation of the stochastic variable as

$$\delta_f \simeq 2\frac{\delta\rho_\xi}{V}\,. \tag{5.1}$$

Similarly, the tensor perturbation $H_T^{(t)}$ is given by the transverse traceless part of the anisotropic pressure of the stochastic variable as

$$\begin{aligned} H^{(t)}_{T\boldsymbol{k}}(\tau) \ &= \ -\frac{\pi\kappa^2}{k^2} \int_x^1 dx' \left(\frac{x}{x'}\right)^{1+\varepsilon} \sqrt{xx'} \\ &\qquad \times \mathrm{Im}[H_\nu^{(1)}(x) H_\nu^{(2)}(x')]\ a^2(\tau')\ p\pi^{(t)}_{\xi\boldsymbol{k}}(x')\,. \end{aligned} \tag{5.2}$$

In this section, we shall evaluate the correlation functions of $\delta\rho_\xi$ and $p\pi_\xi$. In Appendix A, we calculate these correlation functions from the noise kernel. They are expressed in terms of the Wightman Green function for the interaction picture field in momentum space, $G_k^+(\tau_1,\tau_2)$. First we determine the Green function, and then we evaluate the correlation function of $\delta\rho_\xi$ and $p\pi_\xi$.

A. Propagator

As mentioned before, to compute the correlation functions, it is necessary to determine the Wightman function in momentum space,

$$G_k^+(\tau_1,\ \tau_2) \equiv \psi_{f,k}(\tau_1)\psi^*_{f,k}(\tau_2)\,, \tag{5.3}$$

where $\psi_{f,k}(\tau)$ is the mode function of a quantum scalar field, which satisfies the wave equation

$$\psi_{f,k}{}''(\tau) + 2\mathcal{H}\psi_{f,k}{}'(\tau) + \{k^2 + a^2\kappa^2 V\eta_V\}\psi_{f,k}(\tau) = 0\,. \tag{5.4}$$

We solve this equation under the slow-roll condition. Introducing a new variable as $\tilde{\psi}_k(\tau) \equiv a(\tau)\,\psi_{f,k}(\tau)$, this equation is rewritten as

$$\tilde{\psi}_k''(\tau) + [k^2 - \{2-\varepsilon-\eta_V(3-\varepsilon)\}\mathcal{H}^2]\tilde{\psi}_k(\tau) = 0\,, \quad (5.5)$$

where we have used the relation

$$a^2\kappa^2 V = a^2\kappa^2\rho\Big(1-\frac{\varepsilon}{3}\Big) = 3\mathcal{H}^2\Big(1-\frac{\varepsilon}{3}\Big)\,. \quad (5.6)$$

On the sub-Planck scale, we can neglect the term whose magnitude is smaller by the order of $(\kappa H)^2$ than that of the leading term. We also ignore higher-order terms with respect to the slow-roll parameters. So we do not include the time evolution of slow-roll parameters. Under these assumptions, the equation for $\tilde{\psi}$ becomes

$$\frac{d^2}{dx^2}\tilde{\psi}(x) + \Big[1 - \frac{2+3(\varepsilon-\eta_V)}{x^2}\Big]\tilde{\psi}(x) = 0\,, \quad (5.7)$$

where $x \equiv -k\tau$, and we have used $\mathcal{H} \simeq -1/[(1-\varepsilon)\tau]$. The general solution is given by the Hankel functions as

$$\tilde{\psi}_k(\tau) = x^{\frac{1}{2}}\ \Big[\ \tilde{C}H_\beta^{(1)}(x) + \tilde{D}H_\beta^{(2)}(x)\ \Big]\ , \quad (5.8)$$

where $\beta^2 \equiv 9/4 + 3(\varepsilon - \eta_V)$, with two arbitrary integration constants $\tilde{C}$ and $\tilde{D}$. This implies

$$\psi_k(\tau) = \frac{x^{\frac{1}{2}}}{a(\tau)}\ \Big[\ \tilde{C}H_\beta^{(1)}(x) + \tilde{D}H_\beta^{(2)}(x)\ \Big]\ . \quad (5.9)$$

We assume that the mode functions should have the same form as in Minkowski spacetime, i.e.,

$$\psi_k(\tau_i) = \frac{1}{\sqrt{2k}}\ e^{-ik\tau_i}\,, \quad (5.10)$$

when the wavelength is much shorter than the horizon scale, i.e., at very early stage of the universe. This fact may be true in the present gauge rather than the comoving gauge. Then the mode function and the Wightman function in momentum space are given by

$$\psi_k(\tau) = \frac{\sqrt{\pi|\tau|}}{2}\ \frac{a_i}{a(\tau)}\ e^{i\frac{(2\beta+1)\pi}{4}}\ H_\beta^{(1)}(x) \quad (5.11)$$

$$G_k^+(\tau_1,\ \tau_2) = \frac{\pi\sqrt{\tau_1\ \tau_2}}{4}\ \frac{a_i^2}{a_1 a_2}\ H_\beta^{(1)}(x_1)\ H_\beta^{(2)}(x_2)\,. \quad (5.12)$$

Setting $a_i = 1$, we give the scale factor $a(\tau)$ by $a(\tau) = (\tau_i/\tau)^{1+\varepsilon}$. Using this fact, we rewrite the Wightman function as

$$\begin{aligned} G_k^+(\tau_1,\ \tau_2) &= \frac{\pi\sqrt{\tau_1\ \tau_2}}{4}\Big(\frac{\tau_1\tau_2}{\tau_i^2}\Big)^{1+\varepsilon} H_\beta^{(1)}(x_1)\ H_\beta^{(2)}(x_2) \\ &= \frac{\pi}{4}\frac{(x_1x_2)^{\frac{3}{2}}}{k^3}\Big(\frac{\tau_1\tau_2}{\tau_i^2}\Big)^{\varepsilon}(1-\varepsilon)^2 H_i^2\ H_\beta^{(1)}(x_1)\ H_\beta^{(2)}(x_2)\,. \end{aligned} \quad (5.13)$$

Here we have used the relation

$$\tau_i^{-2} = (1-\varepsilon)^2\mathcal{H}_i^2 = (1-\varepsilon)^2 H_i^2\,. \quad (5.14)$$

To compute the correlation functions, it is sufficient to consider the evolution of the Wightman function in the superhorizon region. The behavior of $G_k^+(\tau_1,\ \tau_2)$ in the superhorizon region is summarized in Appendix B in [36].

B. Scalar perturbations

Once the Wightman function is determined, we can compute the correlation function of $\delta\rho_\xi$ from Eq. (A12).

$$\begin{aligned} &\langle\delta\rho_{\xi\boldsymbol{k}}(\tau)\delta\rho_{\xi\boldsymbol{p}}(\tau)\rangle^{(4)} \\ &= \int d^3\boldsymbol{x}_1 d^3\boldsymbol{x}_2 e^{-i(\boldsymbol{k}\cdot\boldsymbol{x}_1+\boldsymbol{p}\cdot\boldsymbol{x}_2)}\langle\xi^0{}_0(x_1)\xi^{0'}{}_{0'}(x_2)\rangle^{(4)}\Big|_{\tau_1,\tau_2=\tau} \\ &= \frac{1}{8}\ \delta(\boldsymbol{k}+\boldsymbol{p})\int d^3\boldsymbol{q} \\ &\quad\times\Big\{(a_1)^{-2}\Big(\partial_{\tau_1}{}^q\partial_{\tau_1}{}^{k-q} - \boldsymbol{q}\cdot(\boldsymbol{k}-\boldsymbol{q})\Big) + \alpha_1^{(2)}V_1^{cl}\kappa^2\Big\} \\ &\quad\times\Big\{(a_2)^{-2}\Big(\partial_{\tau_2}{}^q\partial_{\tau_2}{}^{k-q} - \boldsymbol{q}\cdot(\boldsymbol{k}-\boldsymbol{q})\Big) + \alpha_2^{(2)}V_2^{cl}\kappa^2\Big\} \\ &\quad\times\mathrm{Re}\Big[\ G_q^+(\tau_1,\ \tau_2)G^+_{|\boldsymbol{k}-\boldsymbol{q}|}(\tau_1,\ \tau_2)\Big]\Big|_{\tau_1,\tau_2=\tau}\,, \end{aligned} \quad (5.15)$$

where the number of the superscript (4) represents the power of (κH). We put the momentum superscript on the partial derivative operator. This means, for example, $\partial_{\tau_1}{}^q$ operates only on the Wightman function with the momentum q, $G_q^+(\tau_1,\ \tau_2)$. It is convenient to divide this correlation function into the subhorizon part $I_{\rm sb}(\tau,\boldsymbol{k})$ and the superhorizon part $I_{\rm sp}(\tau,\boldsymbol{k})$, which are defined by

$$\begin{aligned} I_{\rm sb}(\tau,\boldsymbol{k}) \equiv &\int_{q\in[\mathcal{H},\ \infty]} d^3\boldsymbol{q} \\ &\times\Big\{(a_1)^{-2}\Big(\partial_{\tau_1}{}^q\partial_{\tau_1}{}^{k-q} - \boldsymbol{q}\cdot(\boldsymbol{k}-\boldsymbol{q})\Big) + \alpha_1^{(2)}V_1^{cl}\kappa^2\Big\} \\ &\times\Big\{(a_2)^{-2}\Big(\partial_{\tau_2}{}^q\partial_{\tau_2}{}^{k-q} - \boldsymbol{q}\cdot(\boldsymbol{k}-\boldsymbol{q})\Big) + \alpha_2^{(2)}V_2^{cl}\kappa^2\Big\} \\ &\times\mathrm{Re}\Big[\ G_q^+(\tau_1,\ \tau_2)G^+_{|\boldsymbol{k}-\boldsymbol{q}|}(\tau_1,\ \tau_2)\Big]\Big|_{\tau_1,\tau_2=\tau} \end{aligned} \quad (5.16)$$

$$\begin{aligned} I_{\rm sp}(\tau,\boldsymbol{k}) \equiv &\int_{q\in[0,\ \mathcal{H}]} d^3\boldsymbol{q} \\ &\times\Big\{(a_1)^{-2}\Big(\partial_{\tau_1}{}^q\partial_{\tau_1}{}^{k-q} - \boldsymbol{q}\cdot(\boldsymbol{k}-\boldsymbol{q})\Big) + \alpha_1^{(2)}V_1^{cl}\kappa^2\Big\} \\ &\times\Big\{(a_2)^{-2}\Big(\partial_{\tau_2}{}^q\partial_{\tau_2}{}^{k-q} - \boldsymbol{q}\cdot(\boldsymbol{k}-\boldsymbol{q})\Big) + \alpha_2^{(2)}V_2^{cl}\kappa^2\Big\} \\ &\times\mathrm{Re}\Big[\ G_q^+(\tau_1,\ \tau_2)G^+_{|\boldsymbol{k}-\boldsymbol{q}|}(\tau_1,\ \tau_2)\Big]\Big|_{\tau_1,\tau_2=\tau}\,. \end{aligned} \quad (5.17)$$

First we discuss the subhorizon part, $I_{\rm sb}(\tau,\boldsymbol{k})$. Since we consider only the superhorizon mode as the momentum of the external line $(\boldsymbol{k})$, k is much smaller than the horizon scale $\mathcal{H}$. This implies that the momentum of the internal line $\boldsymbol{q}$ in $I_{\rm sb}(\tau,\boldsymbol{k})$, which is larger than $\mathcal{H}$, is much larger than the

external momentum, k. Hence, we can approximate $|\boldsymbol{q}-\boldsymbol{k}|$ as q. So $I_{\rm sb}(\tau,\boldsymbol{k})$ depends only on τ, and then

$$I_{\rm sb}(\tau,\boldsymbol{k}) = I_{\rm sb}(\tau) = \frac{1}{k^3}k^3 I_{\rm sb}(\tau) \propto \frac{1}{k^3}(-k\tau)^3\,. \quad (5.18)$$

Here we have separated the scale invariant part of k^{-3}. Since the remaining part, $k^3 I_{\rm sb}(\tau)$, must be a function of $-k\tau$, we find that even the leading part of $I_{\rm sb}(\tau,\boldsymbol{k})$ decays as $(-k\tau)^3$. Hence we can neglect the contribution from the subhorizon region.

Consequently, the correlation function of $\delta\rho_{\xi\boldsymbol{k}}(\tau)$ is evaluated only in the superhorizon region as

$$\langle \delta\rho_{\xi\boldsymbol{k}}(\tau)\delta\rho_{\xi\boldsymbol{p}}(\tau)\rangle^{(4)} \simeq \frac{1}{8}\,\delta(\boldsymbol{k}+\boldsymbol{q})\, I_{\rm sp}(\tau,\,\boldsymbol{k})\,. \quad (5.19)$$

As seen in Appendix B in [36], in the superhorizon region, the Wightman function $G_k^+(\tau_1,\tau_2)$ is approximated as

$$\begin{aligned} G_k^+(\tau_1,\,\tau_2) &\simeq \frac{1}{2}\frac{\sqrt{\tau_1\tau_2}}{a_1 a_2}\,(x_1 x_2)^{-\beta} \\ &\simeq \frac{1}{2}\frac{(\tau_1\tau_2)^{\frac{1}{2}-\beta}}{a_1 a_2}\,k^{-2\beta}\,. \end{aligned} \quad (5.20)$$

Substituting this expression into the definition of $I_{\rm sp}(\tau,\boldsymbol{k})$ and neglecting the sub-leading terms w.r.t. the slow-roll parameters, we obtain $I_{\rm sp}(\tau,\,\boldsymbol{k})$ as

$$\begin{aligned} I_{\rm sp}(\tau,\boldsymbol{k}) &= \left(\eta_V V^{cl}\kappa^2\right)^2 \frac{1}{4}\frac{|\tau|^{2-4\beta}}{a(\tau)^4} \\ &\times \int_{q\in[0,\,\mathcal{H}]} \frac{d^3\boldsymbol{q}}{q^{3+2(\varepsilon-\eta_V)}\,|\boldsymbol{k}-\boldsymbol{q}|^{3+2(\varepsilon-\eta_V)}}\,. \end{aligned} \quad (5.21)$$

Here we encounter the so-called infrared divergence problem. In the long wave limit ($q\to 0$), the integrand is approximately $q^{-[3+2(\varepsilon-\eta_V)]}$. Then this integral could be divergent depending on the signature of $(\varepsilon-\eta_V)$[68]. This is the infrared (IR) problem, which sometimes appears in the quantum field theory in an inflationary universe. When we use the scale invariant power spectrum, in general we find this divergence on the loop corrections. Here, introducing the cut off by the initial horizon scale, we just neglect the effects from the long wave modes whose comoving lengths are larger than the initial horizon scale $a_i H_i$. We tentatively discuss this IR problem in Sec.VII and elaborate this problem in [69].

After introducing the cut off H_i and integrating over the internal momentum $\boldsymbol{q}$, we obtain a finite result. Using the loop integral Eq. (B4), whose detailed derivation is given in Appendix C, we find $I_{\rm sp}(\tau,\boldsymbol{k})$ as

$$\begin{aligned} & I_{\rm sp}(\tau,\boldsymbol{k}) \\ &= \left(\eta_V V^{cl}\kappa^2\right)^2 \frac{\pi}{k^3} H(\tau)^4 x^{-4(\varepsilon-\eta_V)} \\ &\times \Big[\, \frac{1}{3} - \frac{1-(H_i/k)^{-2(\varepsilon-\eta_V)}}{2(\varepsilon-\eta_V)} - \frac{1}{3}\Big(\frac{k}{\mathcal{H}}\Big)^{3+4(\varepsilon-\eta_V)} \Big]\,. \end{aligned} \quad (5.22)$$

Substituting this result into Eq. (5.19), we obtain the correlation function of $\delta\rho_{\xi\boldsymbol{k}}(\tau)$ as

$$\begin{aligned} & \langle \delta\rho_{\xi\boldsymbol{k}}(\tau)\delta\rho_{\xi\boldsymbol{p}}(\tau)\rangle^{(4)} \\ &\simeq \frac{\pi}{8}\,\frac{\{\kappa H(\tau)\}^4}{k^3}\Big(\eta_V V^{cl}\Big)^2\, x^{4(\eta_V-\varepsilon)}\,\delta(\boldsymbol{k}+\boldsymbol{p}) \\ &\times \Big[\, \frac{1}{3} - \frac{1-(H_i/k)^{-2(\varepsilon-\eta_V)}}{2(\varepsilon-\eta_V)} - \frac{1}{3}\Big(\frac{k}{\mathcal{H}}\Big)^{3+4(\varepsilon-\eta_V)} \Big]\,. \end{aligned} \quad (5.23)$$

If $\varepsilon > \eta_V$, it diverges when we remove the cut-off H_i.

C. Tensor perturbations

Next we calculate the correlation function of the transverse traceless part of the anisotropic pressure of the stochastic variable, $p\pi_{\xi\boldsymbol{k}}(\tau)$, which is given by Eq.(A17) in Appendix B, as

$$\begin{aligned} & \langle p\pi_{\xi\boldsymbol{k}}(\tau_1) e^i{}_j(\boldsymbol{k}) p\pi_{\xi\boldsymbol{p}}(\tau_2) e^j{}_i(\boldsymbol{p})\rangle^{(4)} \\ &= \frac{1}{4(a_1 a_2)^2}\,\delta(\boldsymbol{k}+\boldsymbol{p}) \int d^3\boldsymbol{q}\,\Big(q^2 - \frac{(\boldsymbol{k}\cdot\boldsymbol{q})^2}{k^2}\Big)^2 \\ &\qquad \times {\rm Re}\Big[\, G_q^+(\tau_1,\,\tau_2) G^+_{|\boldsymbol{k}-\boldsymbol{q}|}(\tau_1,\,\tau_2)\,\Big]\,. \end{aligned} \quad (5.24)$$

For the tensor perturbations, we also divide the correlation functions into the subhorizon part $J_{\rm sb}(\tau_1,\tau_2,\boldsymbol{k})$ and the superhorizon part $J_{\rm sp}(\tau_1,\tau_2,\boldsymbol{k})$, which are defined as

$$\begin{aligned} & J_{\rm sb}(\tau_1,\tau_2,\boldsymbol{k}) \\ &\equiv \theta(\tau_1-\tau_2)\int_{q\in[\mathcal{H}_2,\infty]} d^3\boldsymbol{q}\,\Big(q^2 - \frac{(\boldsymbol{k}\cdot\boldsymbol{q})^2}{k^2}\Big)^2 \\ &\qquad \times {\rm Re}\Big[\, G_q^+(\tau_1,\,\tau_2) G^+_{|\boldsymbol{k}-\boldsymbol{q}|}(\tau_1,\,\tau_2)\,\Big] \\ &+ \theta(\tau_2-\tau_1)\int_{q\in[\mathcal{H}_1,\infty]} d^3\boldsymbol{q}\,\Big(q^2 - \frac{(\boldsymbol{k}\cdot\boldsymbol{q})^2}{k^2}\Big)^2 \\ &\qquad \times {\rm Re}\Big[\, G_q^+(\tau_1,\,\tau_2) G^+_{|\boldsymbol{k}-\boldsymbol{q}|}(\tau_1,\,\tau_2)\,\Big] \end{aligned} \quad (5.25)$$

$$\begin{aligned} & J_{\rm sp}(\tau_1,\tau_2,\boldsymbol{k}) \\ &\equiv \theta(\tau_1-\tau_2)\int_{q\in[H_i,\mathcal{H}_2]} d^3\boldsymbol{q}\,\Big(q^2 - \frac{(\boldsymbol{k}\cdot\boldsymbol{q})^2}{k^2}\Big)^2 \\ &\qquad \times {\rm Re}\Big[\, G_q^+(\tau_1,\,\tau_2) G^+_{|\boldsymbol{k}-\boldsymbol{q}|}(\tau_1,\,\tau_2)\,\Big] \\ &+ \theta(\tau_2-\tau_1)\int_{q\in[H_i,\mathcal{H}_1]} d^3\boldsymbol{q}\,\Big(q^2 - \frac{(\boldsymbol{k}\cdot\boldsymbol{q})^2}{k^2}\Big)^2 \\ &\qquad \times {\rm Re}\Big[\, G_q^+(\tau_1,\,\tau_2) G^+_{|\boldsymbol{k}-\boldsymbol{q}|}(\tau_1,\,\tau_2)\,\Big]\,. \end{aligned} \quad (5.26)$$

Note that to compute the correlation function for the tensor perturbation, $H^{(t)}_{T\boldsymbol{k}}(\tau)$ at conformal time τ, it is necessary

to consider the correlation function of $p\pi_{\xi \boldsymbol{k}}$ for two different times τ_1 and τ_2. This is because, as seen from Eq. (5.2), the expression of $H_T^{(t)}$ includes the time integral. Therefore, there are two different comoving horizon scales corresponding to the different times τ_1 and τ_2. For the same reason as in the case of the scalar perturbations, we have introduced the IR cut-off H_i. Nevertheless, we can see later, for the tensor perturbations, even if we remove the IR cut-off, the loop corrections remains finite.

By virtue of the same argument as that presented in the scalar perturbations, $J_{\rm sb}(\tau_1,\tau_2,\boldsymbol{k})$ contains only the decaying modes. To show this, note that if either $-k\tau_1$ or $-k\tau_2$ is larger than unity, it does not produce cumulative contributions because of the oscillation of the Hankel function in subhorizon region, as mentioned in Eq. (4.24). Hence, it is sufficient to consider only the case where both $-k\tau_1$ and $-k\tau_2$ are smaller than unity. If $\tau_1 \geq \tau_2$, then the inner momentum q is larger than $\mathcal{H}_2 \simeq -1/\tau_2$. Hence, this implies that q is larger than k. Approximating $|\boldsymbol{k}-\boldsymbol{q}|$ as q, we find that $J_{\rm sb}(\tau_1,\tau_2,\boldsymbol{k})$ contains only the decaying mode as we have shown in the scalar perturbations. The same discussion is valid also in case of $\tau_2 \geq \tau_1$. Hence, in order to compute the correlation function of $p\pi_\xi^{(t)}$, it is sufficient to consider only the contribution from the superhorizon region, $J_{\rm sp}(\tau_1,\tau_2,\boldsymbol{k})$.

As with the case of scalar perturbations, substituting the approximation of the Wightman function in the superhorizon region into the definition of $J_{\rm sp}(\tau_1,\tau_2,\boldsymbol{k})$, we find the contribution from the superhorizon region as

$$\begin{aligned} J_{\rm sp}(\tau_1,\tau_2,\boldsymbol{k}) &\equiv \frac{1}{4}\frac{(\tau_1\tau_2)^{1-2\beta}}{(a_1a_2)^2} \\ &\cdot\Big[\,\theta(\tau_1-\tau_2)\int_{q\in[H_i,\mathcal{H}_2]} d^3\boldsymbol{q}\left(q^2-\frac{(\boldsymbol{k}\cdot\boldsymbol{q})^2}{k^2}\right)^2 \\ &\qquad\times\frac{1}{q^{3+2(\varepsilon-\eta_V)}\ |\boldsymbol{q}-\boldsymbol{k}|^{3+2(\varepsilon-\eta_V)}} \\ &\quad+\theta(\tau_2-\tau_1)\int_{q\in[H_i,\mathcal{H}_1]} d^3\boldsymbol{q}\left(q^2-\frac{(\boldsymbol{k}\cdot\boldsymbol{q})^2}{k^2}\right)^2 \\ &\qquad\times\frac{1}{q^{3+2(\varepsilon-\eta_V)}\ |\boldsymbol{q}-\boldsymbol{k}|^{3+2(\varepsilon-\eta_V)}}\Big]\,. \end{aligned} \tag{5.27}$$

This loop integral is given by Eq. (B5) in Appendix C. It implies

$$\begin{aligned} &J_{\rm sp}(\tau_1,\ \tau_2,\boldsymbol{k}) \\ &\simeq \frac{8\pi}{15}\,H_k^4 k\Big[\theta(x_2-x_1)\,x_1^{2\eta_V}x_2^{-1+4\varepsilon-2\eta_V} \\ &\qquad+\theta(x_1-x_2)\,x_1^{-1+4\varepsilon-2\eta_V}x_2^{2\eta_V}-\frac{3}{4}(x_1x_2)^{2\eta_V}\Big]\,, \end{aligned} \tag{5.28}$$

where we have used $H(\tau)^2x^{-2\varepsilon} = H_k^2$. To derive this relation, we have taken the limit of $H_i \to 0$. No divergence appears. It means that the cut-off for the infrared region is not necessary. It is interesting to note that scalar perturbations have the infrared divergence, but tensor perturbations do not suffer from the infrared problem. In Sec.VII, we discuss the origin of this infrared divergence and the reason why only tensor perturbations does not contain it.

As a result, we obtain the correlation function for the tensor part of the anisotropic pressure as

$$\begin{aligned} &\langle p\pi_{\xi\boldsymbol{k}}(\tau_1)e^i{}_j(\boldsymbol{k})\ p\pi_{\xi\boldsymbol{p}}(\tau_2)e^j{}_i(\boldsymbol{p})\rangle^{(4)} \\ &\simeq \frac{1}{4}\,(a_1a_2)^{-2}\,\delta(\boldsymbol{k}+\boldsymbol{p})\ J_{\rm sp}(\tau_1,\tau_2,\boldsymbol{k}) \\ &\simeq \frac{2\pi}{15}\,\frac{H_k^4k}{(a_1a_2)^2}\,\delta(\boldsymbol{k}+\boldsymbol{p})\Big[\theta(x_2-x_1)x_1^{2\eta_V}x_2^{-1+4\varepsilon-2\eta_V} \\ &\qquad+\theta(x_1-x_2)x_1^{-1+4\varepsilon-2\eta_V}x_2^{2\eta_V}-\frac{3}{4}(x_1x_2)^{2\eta_V}\Big]\,. \end{aligned} \tag{5.29}$$

In our computation, we have neglected the loop corrections from the tensor perturbations, because the amplitude of power spectrum for the tensor perturbations is smaller than that for the scalar perturbations by order of the slow-roll parameter ε.

VI. LOOP CORRECTIONS TO THE CORRELATION FUNCTIONS

As shown in Eqs. (5.1) and (5.2), the leading parts of the correlation function of the density perturbation in a flat-slicing δ_f and of the tensor perturbation $H_T^{(t)}$ are determined by the stochastic variables $\delta\rho_\xi$ and $p\pi_\xi^{(t)}$. We then have calculated the correlation functions of $\delta\rho_\xi$ and $p\pi_\xi$ for the noise kernel. Combining these results, we shall evaluate the correlation functions of δ_f and $H_T^{(t)}$.

A. Scalar perturbations

To evaluate the correlation function of the density perturbation in flat-slicing δ_f, focusing on the proper nonlinear effects, we neglect the contribution from the product of linear perturbations. This density perturbation is related to the curvature perturbation in uniform slicing ζ by Eq. (4.4). Hence, once we find the loop corrections to δ_f, we also obtain the loop corrections to ζ. The curvature perturbation ζ is proportional to the gravitational potential in the late time of the universe and it is directly related to the fluctuation of the temperature of CMB. That is why it is important for us to consider this gauge-invariant variable among scalar perturbations.

From Eq. (5.1) and (5.23), the loop corrections to the correlation function of the density perturbation are given by

$$\begin{aligned} &\langle\delta_{f\boldsymbol{k}}(\tau)\delta_{f\boldsymbol{p}}(\tau)\rangle^{(4)} \\ &\simeq \frac{\pi}{2}\frac{(\kappa H_k)^4}{k^3}\eta_V^2(-k\tau)^{4\eta_V}\ \delta(\boldsymbol{k}+\boldsymbol{p}) \\ &\qquad\times\Big[\frac{1}{3}-\frac{1-(k/H_i)^{2(\varepsilon-\eta_V)}}{2(\varepsilon-\eta_V)}\Big]\,, \end{aligned} \tag{6.1}$$

where we have used the relation of $H(\tau)^2x^{-2\varepsilon} = H_k^2$. In our previous work [36], we showed that when we solve explicitly the Einstein- Langevin equation [24], which includes

an iterative aspect, it disturbs the constant evolution of ζ in superhorizon region. Since this is a problem of the way to quantize the scalar field and the gravitational field, the same effects may exist in the present loop corrections. Hence, we restrict our discussions to the case when $\eta_V \log x$ is smaller than unity, i.e., we assume that $(-k\tau)^{4\eta_V} \approx 1$. Then, we find the correlation function of the density perturbation δ_f as

$$\begin{aligned}&\langle \delta_{f\boldsymbol{k}}(\tau)\delta_{f\boldsymbol{p}}(\tau)\rangle^{(4)} \\ &\simeq \frac{\pi}{2}\frac{(\kappa H_k)^4}{k^3}\eta_V^2 \delta(\boldsymbol{k}+\boldsymbol{p})\left\{\frac{1}{3} - \frac{1-(k/H_i)^{2(\varepsilon-\eta_V)}}{2(\varepsilon-\eta_V)}\right\}.\end{aligned} \tag{6.2}$$

Taking into account Eq. (4.4), we obtain the loop corrections to the correlation function of the curvature perturbation in uniform density slicing ζ as

$$\begin{aligned}&\langle \zeta_{\boldsymbol{k}}(\tau)\zeta_{\boldsymbol{p}}(\tau)\rangle^{(4)} \\ &\simeq \frac{\pi}{8}\,\frac{(\kappa H_k)^4}{k^3}\,\left(\frac{\eta_V}{\varepsilon}\right)^2 \delta(\boldsymbol{k}+\boldsymbol{p})\left\{\frac{1}{3} - \frac{1-(k/H_i)^{2(\varepsilon-\eta_V)}}{2(\varepsilon-\eta_V)}\right\}.\end{aligned} \tag{6.3}$$

The final result depends on the initial Hubble horizon scale, $\mathcal{H}_i = H_i$, which is introduced to remove the infrared divergence. The case with $2|\varepsilon - \eta_V|\log(k/\mathcal{H}_i) < 1$ is particularly interesting. This, in other words, corresponds to the case of $N_k < 1/2|\varepsilon - \eta_V|$, where $N_k \simeq \log(k/\mathcal{H}_i)$ is the e-folding from the beginning of inflation to the horizon crossing time. In this case, this correlation function is approximated as

$$\begin{aligned}&\langle \zeta_{\boldsymbol{k}}(\tau)\zeta_{\boldsymbol{p}}(\tau)\rangle^{(4)} \\ &\simeq \frac{\pi}{8}\,\frac{(\kappa H_k)^4}{k^3}\,\left(\frac{\eta_V}{\varepsilon}\right)^2 \delta(\boldsymbol{k}+\boldsymbol{p})\,\left(\frac{1}{3}+N_k\right).\end{aligned} \tag{6.4}$$

Note that there appears the logarithmic corrections. These results imply that although the one-loop correction is suppressed by $(\kappa H_k)^4$ and is smaller by the order of the $(\kappa H_k)^2$ than tree-level effects, it is amplified by the e-folding N_k from the initial time to the horizon crossing time, which can become large contrary to the e-folding from the horizon crossing time to the end of the inflation. However note that this amplification is derived by introducing the IR cut-off and the obtained loop corrections significantly depend on the choice of the IR cut-off.

B. Tensor perturbations

The tensor perturbation $H_T^{(t)}$ is related to the source term $p\pi_\xi^{(t)}$, and the correlation function of $p\pi_\xi^{(t)}$ is given by Eq. (5.29). Then, integrating over $x = -k\tau$, we obtain the correlation function and the amplitude of the tensor perturbation, which could be amplified by the quantum effect of a scalar field. To integrate over x, it is helpful to note the asymptotic behaviour of the Hankel function when the argument x is smaller than unity. As summarized in Appendix B in [36], the part of the integrand is approximated as

$$\mathrm{Im}\Big[H_\nu{}^{(1)}(x)\;H_\nu{}^{(2)}(x_1)\Big] \simeq -\frac{2}{3\pi}\Big[\{\Big(\frac{x_1}{x}\Big)^\nu - \Big(\frac{x}{x_1}\Big)^\nu\Big] \tag{6.5}$$

Using this approximation, we find the loop corrections to the correlation function of the tensor perturbations as

$$\begin{aligned}&\langle H_{T\boldsymbol{k}}^{(t)}(\tau)e^i{}_j(\boldsymbol{k})\;H_{T\boldsymbol{p}}^{(t)}(\tau)e^j{}_i(\boldsymbol{p})\rangle^{(4)} \\ &\simeq \frac{\pi}{135}\frac{(\kappa H_k)^4}{k^3}\;\delta(\boldsymbol{k}+\boldsymbol{p})\;\Big(\;\frac{7}{6} - 15\;x^{2+2\eta_V}\Big)\,.\end{aligned} \tag{6.6}$$

It is interesting to note that there is no dependence on the infrared cut off H_i in the tensor perturbations. Furthermore, the tensor perturbations are divergence free in the superhorizon region. We shall discuss the reason in the next section. In our computation, we have neglected the loop corrections from the tensor perturbations, since the amplitude of the tensor perturbations are smaller than that of the scalar perturbations by order of the slow-roll parameter ε.

VII. DISCUSSIONS

Using the Einstein-Langevin equation proposed in [24], we calculate the loop corrections to the scalar perturbations and the tensor perturbations, which are amplified through the nonlinear interaction between the scalar field and the gravitational field. Here we discuss the origin of the amplification.

When we consider the loop corrections in inflationary universe, there are two different divergences. One is the ultraviolet (UV) divergence. Since this divergence is originated by short wave modes, such divergence also appears in the quantum field theory in a Minkowski background. In inflationary spacetime, there exists another divergence, which is not found in Minkowski spacetime. This is the IR divergence. To avoid this IR divergence, we have introduced the cut-off at the initial Hubble horizon size. Then the amplitude of the one-loop corrections to the curvature perturbation ζ is amplified by the e-folding from the initial time of inflation to the horizon crossing time, i.e., the logarithmic correction. If this is true, this amplification may make it possible to detect these loop corrections. Then it will be a great help to clarify the fundamental properties of an inflation model.

So far we have several discussions about this logarithmic corrections due to IR divergence. Early works about this problem are done by Boyanovsky, de Vega and Sanchez [64–67]. They calculated one loop corrections by light scalar and fermion fields to the inflaton potential, and also evaluated those by the gauge invariant curvature and tensor perturbations. They found that there appear the IR enhancements both in the scalar field corrections and curvature perturbations, while both fermion corrections and tensor perturbations do not exhibit IR divergences. Weinberg also pointed that the loop corrections to the primordial perturbations behave at most logarithmic [9, 10]. Afterward Sloth considered the loop corrections to the fluctuation of the scalar field in flat-slicing [11, 12]. To avoid IR divergence, he introduced the

cut off by the initial horizon scale. As a result, he found that the loop correction is amplified by the e-folding from the initial time of inflation to the horizon crossing time, which is also found in this paper from the analysis based on stochastic gravity. Following Sloth, Seery readdressed this problem more carefully [13, 14]. In particular, he analysed the evolution in the superhorizon region using the δN formula [18, 53], and improved his results. In this paper, we have computed the loop corrections by stochastic gravity, and found the similar logarithmic corrections for scalar perturbations. The same logarithmic behaviours have been found in other interacting systems [59–61]. However, we should note that the IR problem requires the careful treatment and the way to evaluate IR effects is controversial [62, 63].

Recently, Lyth has claimed that, to avoid the assumptions on unknown parts of the universe, the calculation about loop corrections should be done inside a comoving box, whose size L is not too much bigger than the present horizon scale [15]. The IR corrections are significantly reduced, although we still find the logarithmic behaviours. Furthermore, Bartolo et al. claimed that a stochastic approach plays a crucial role to deal with this problem [17]. In relation to their insists, we should stress our interesting results. That is, although the scalar perturbations are amplified by the logarithmic corrections, the tensor perturbations are not. Even if we remove the IR cut-off, the IR divergence does not appear in tensor perturbations. This difference between the scalar and tensor perturbations seems to be related to the origin of these logarithmic corrections.

To consider the origin of this logarithmic corrections due to the IR cut-off, we first mention the prediction in stochastic inflation[40–42, 44–51]. In stochastic inflation, the long wave mode k with $k < aH$ of the scalar field couples to the short wave mode k with $k > aH$ through the nonlinear self-interaction of the scalar field. Then the long wave modes are affected by the quantum fluctuation of the short wave modes. As a result, the long wave modes come to show stochastic behavior. This stochastic behavior of the long wave modes affects the background quantities.

In our case, due to the nonlinear interaction between the gravitational field and the scalar field, the long wave modes and the background quantities come to show the stochastic behavior. Since the scalar perturbations are defined as the deviation from the background quantities, the stochastic fluctuation of the background quantities affect the behavior of the perturbed variables. As a result, it induces the logarithmic secular evolution of the perturbed variables. On the other hand, there are no background tensor modes, and then the one loop corrections to the tensor perturbation can avoid being affected by the background stochastic fluctuations. Furthermore, although in this paper, as the simplest step for treating the IR divergence, we have simply neglected the long wave modes with $-k\mathcal{H}_i > 1$, the infrared modes require the more careful treatment. We will propose the way to regularize the IR corrections in [69].

There is another notable difference between the scalar and the tensor perturbations. As pointed out in our previous work, in the present approach of stochastic gravity the longitudinal part of gravitational field is included iteratively. This affects the behavior of perturbations in the superhorizon region. In particular, the curvature perturbation deviates from constant when the e-folding from the horizon crossing time exceeds the definite critical value ($|\text{slow-roll parameter}|^{-1}$). Since this is the problem of the way to quantize the gravitational field and the matter field, the loop corrections to the scalar perturbations are also influenced by the nonexistence of the longitudinal part of the quantized gravitational field. In fact, as shown in Eq. (6.1), the one loop correction to the curvature perturbation ζ evolves as $x^{4\eta_V}$ in the superhorizon region. On the other hand, as shown in Eq. (6.6), the tensor perturbations do not decay. This means that even if we use the Einstein-Langevin equation in the present iterative way, it does not affect the one loop correction to the tensor perturbations.

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Appendix A: Computations of the Noise kernel

In Appendix A, we calculate the noise kernel, which is defined by Eq. (2.2) and (2.3). As shown in Sec.VI, we have to compute the correlation function of the density perturbation and the transverse traceless part of the anisotropic pressure of the stochastic variable ξ_{ab}. Then, we compute the $(0,0,0',0')$ component and the transverse traceless part of the (i,j,k',l') component of the noise kernel, i.e., $F^{0}{}_{0}{}^{0'}{}_{0'}(x_1,x_2)$ and $F^{i}{}_{j}{}^{k'}{}_{l'}(x_1,x_2)$. Note that the noise kernel is computed from the quantum fluctuation of the

scalar field on the background spacetime. Decomposing the scalar field into $\phi = \phi_{cl} + \psi$, the energy momentum tensor

$$T_{ab} = \nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}[\nabla_c\phi\nabla^c\phi + 2V(\phi)] \tag{A1}$$

is expressed as the classical part and the fluctuation part as follows:

$$\begin{aligned} T_{ab} \;=&\; T^{(cl)}_{ab} + \delta T_{ab}\,; \\ & T^{(cl)}_{ab} \equiv \delta^0{}_a\delta^0{}_b a^2\dot{\phi}^2_{cl} + \frac{1}{2}g_{ab}\dot{\phi}^2_{cl} - g_{ab}V(\phi_{cl}) \qquad \text{(A2)} \\ & \delta T_{ab} \equiv (\delta^0{}_a\nabla_b\psi + \delta^0{}_b\nabla_a\psi + \eta_{ab}\nabla_0\psi)a\dot{\phi}_{cl} + \nabla_a\psi\nabla_b\psi - \frac{1}{2}g_{ab}\nabla_c\psi\nabla^c\psi - g_{ab}V(\phi_{cl})\sum_{m=1}^{\infty}\frac{\alpha^{(m)}}{m!}(\kappa\psi)^m\,. \qquad \text{(A3)} \end{aligned}$$

The noise kernel, which represents the fluctuation of the energy-momentum tensor, can be expressed in terms of δT_{ab}. In fact, substituting this decomposed energy-momentum tensor into the definition of $F_{abc'd'}(x_1,x_2)$, we can express the two-point function $F_{abc'd'}(x_1,x_2)$ as

$$\begin{aligned} & F_{abc'd'}(x_1,x_2) \\ &= \langle\widehat{\delta T}_{ab}(x_1)\widehat{\delta T}_{c'd'}(x_2)\rangle - \langle\widehat{\delta T}_{ab}(x_1)\rangle\langle\widehat{\delta T}_{c'd'}(x_2)\rangle \\ &= a_1a_2\dot{\phi}_{cl,1}\dot{\phi}_{cl,2}\,[\,\delta^0{}_a\delta^0{}_{c'}G^H_{;bd'} + \delta^0{}_a\delta^0_{d'}G^H_{;bc'} + \delta^0{}_b\delta^0_{c'}G^H_{;ad'} + \delta^0{}_b\delta^0_{d'}G^H_{;ac'} \\ &\qquad + \eta_{c'd'}(\delta^0{}_aG^H_{;b0'} + \delta^0{}_bG^H_{;a0'}) + \eta_{ab}(\delta^{0'}_{c'}G^H_{;0d'} + \delta^{0'}_{d'}G^H_{;0c'}) + \eta_{ab}\eta_{c'd'}G^H_{;00'}] \\ &\quad + G^H_{;ac'}G^H_{;bd'} + G^H_{;ad'}G^H_{;bc'} - (a_1)^2\eta_{ab}G^H_{;ec'}G^{H;e}_{;d'} - (a_2)^2\eta_{c'd'}G^H_{;af'}G^{H;f'}_{;b} + \frac{(a_1a_2)^2}{2}\eta_{ab}\eta_{c'd'}G^H_{;ef'}G^{H;ef'} \\ &\quad - (a_1)^2\eta_{ab}V_{cl,1}\Big[a_2\dot{\phi}_{cl,2}\,\tilde{\alpha}^{(1)}_1\kappa(\delta^{0'}_{c'}G^H_{;d'} + \delta^{0'}_{d'}G^H_{;c'} + \eta_{c'd'}G^H_{;0'}) + \tilde{\alpha}^{(2)}_1\kappa^2\Big(G^H_{;c'}G^H_{;d'} - \frac{(a_2)^2}{2}\,\eta_{c'd'}G^H_{;f'}G^{H;f'}\Big)\Big] \\ &\quad - (a_2)^2\eta_{c'd'}V_{cl,2}\Big[a_1\dot{\phi}_{cl,1}\,\tilde{\alpha}^{(1)}_2\kappa(\delta^0{}_aG^H_{;b} + \delta^0{}_bG^H_{;a} + \eta_{ab}G^H_{;0}) + \tilde{\alpha}^{(2)}_2\kappa^2\Big(G^H_{;a}G^H_{;b} - \frac{(a_1)^2}{2}\,\eta_{ab}G^H_{;e}G^{H;e}\Big)\Big] \\ &\quad + (a_1a_2)^2\eta_{ab}\eta_{c'd'}V_{cl,1}V_{cl,2}\Big[\tilde{\alpha}^{(1)}_1\tilde{\alpha}^{(1)}_2\kappa^2G^H + \frac{\tilde{\alpha}^{(2)}_1\tilde{\alpha}^{(2)}_2}{2!}(\kappa^2G^H)^2 + \frac{\tilde{\alpha}^{(3)}_1\tilde{\alpha}^{(3)}_2}{3!}(\kappa^2G^H)^3 + O\Big((\varepsilon^{1/2}_{SR}\kappa H)^8\Big)\Big]\,, \end{aligned} \tag{A4}$$

where $G^H \equiv \langle\Omega|\hat{\psi}_H(x_1)\hat{\psi}_H(x_2)|\Omega\rangle$ is the Wightman Green function for the interacting system, which is defined as the two-point function of the Heisenberg field $\hat{\psi}_H$. Here we have redefined the coefficients $\tilde{\alpha}^m$, including the divergent part $G^H(x,x)$ as follows:

$$\begin{aligned} \tilde{\alpha}^{(1)}_x &\equiv \alpha^{(1)}_x + \frac{\alpha^{(3)}_x}{2}\kappa^2G^H_{xx} + \frac{\alpha^{(5)}_x}{8}\Big\{\kappa^2G^H_{xx}\Big\}^2 + O\Big((\varepsilon^{1/2}_{SR}\kappa H)^6\Big) \\ \tilde{\alpha}^{(2)}_x &\equiv \alpha^{(2)}_x + \frac{\alpha^{(4)}_x}{2}\kappa^2G^H_{xx} + O(\varepsilon^3_{SR}(\kappa H)^4) \\ \tilde{\alpha}^{(3)}_x &= \alpha^{(3)}_x + O\Big((\varepsilon^{1/2}_{SR}\kappa H)^2\Big)\,. \end{aligned} \tag{A5}$$

Strictly speaking, we have to renormalize these divergent terms into the coefficients of the potential $V(\phi)$. In this paper, we assume that these divergent parts are removed by an appropriate renormalization procedure. So the finite part of these radiative correction terms is much smaller than the leading term among the coefficients $\tilde{\alpha}^{(m)}$. Here we neglect them and approximate $\tilde{\alpha}^{(m)}$ as $\alpha^{(m)}$.

In our previous work [36], we discussed the linear perturbations which are proportional to $(\kappa H)^2$. In this paper, we consider the leading loop corrections which are proportional to $(\kappa H)^4$. The self-interaction part contributes to the effective action from the order $(\kappa H)^6$, which is depicted in Fig. 2 (3). In these diagrams, the solid line represents the interacting picture field which satisfies the equation

$$[\partial^2_0 + (D-2)\mathcal{H}\partial_0 - \nabla^2 + a^2V''(\phi_{cl})]\,\psi_f(x) = 0\,. \tag{A6}$$

When we compute the effective action up to the order of $(\kappa H)^4$, we can replace the Wightman function for the Heisenberg field $G^H(x_1,x_2)$ to the Wightman function for the interaction picture field $G^+(x_1,x_2)$. Then, the parts of $F_{abc'd'}(x_1,x_2)$, whose

orders are $(\kappa H)^2$ and $(\kappa H)^4$, are given by

$$
\begin{aligned}
F^{(2)}_{abc'd'}(x_1,x_2) \;=\;& a_1 a_2 \dot\phi_{cl,1}\dot\phi_{cl,2}\,[\,\delta^0{}_a\delta^0{}_{c'}G^+_{;bd'}+\delta^0{}_a\delta^0_{d'}G^+_{;bc'}+\delta^0{}_b\delta^0_{c'}G^+_{;ad'}+\delta^0{}_b\delta^0_{d'}G^+_{;ac'}\\
&+\eta_{c'd'}(\delta^0{}_aG^+_{;b0'}+\delta^0{}_bG^+_{;a0'})+\eta_{ab}(\delta^{0'}{}_{c'}G^+_{;0d'}+\delta^{0'}{}_{d'}G^+_{;0c'})+\eta_{ab}\eta_{c'd'}G^+_{;00'}]\\
&-(a_1)^2\eta_{ab}V_{cl,1}a_2\dot\phi_{cl,2}\;\alpha_1^{(1)}\;\kappa(\delta^{0'}_{c'}\;G^+_{;d'}+\delta^{0'}_{d'}\;G^+_{;c'}+\eta_{c'd'}\;G^+_{;0'})\\
&-(a_2)^2\eta_{c'd'}V_{cl,2}a_1\dot\phi_{cl,1}\alpha_2^{(1)}\;\kappa(\delta^0{}_a\;G^+_{;b}+\delta^0{}_b\;G^+_{;a}+\eta_{ab}\;G^+_{;0})\\
&+(a_1a_2)^2\eta_{ab}\eta_{c'd'}V_{cl,1}V_{cl,2}\;\alpha_1^{(1)}\alpha_2^{(1)}\kappa^2G^+
\end{aligned}
\tag{A7}
$$

and

$$
\begin{aligned}
F^{(4)}_{abc'd'}(x_1,x_2) \;=\;& G^+_{;ac'}G^+_{;bd'}+G^+_{;ad'}G^+_{;bc'}-(a_1)^2\eta_{ab}G^+_{;ec'}G^{;e}{}_{;d'}-(a_2)^2\eta_{c'd'}G^+_{;af'}G^{+\;;f'}_{;b}+\frac{(a_1a_2)^2}{2}\eta_{ab}\eta_{c'd'}G^+_{;ef'}G^{+;ef'}\\
&-(a_1)^2\eta_{ab}V_{cl,1}\alpha_1^{(2)}\kappa^2\left\{\;G^+_{;c'}G^+_{;d'}-\frac{1}{2}\;(a_2)^2\eta_{c'd'}G^+_{;f'}G^{+;f'}\;\right\}\\
&-(a_2)^2\eta_{c'd'}V_{cl,2}\alpha_2^{(2)}\kappa^2\left\{\;G^+_{;a}G^+_{;b}-\frac{1}{2}\;(a_1)^2\eta_{ab}G^+_{;e}G^{+;e}\;\right\}\\
&+(a_1a_2)^2\eta_{ab}\eta_{c'd'}V_{cl,1}V_{cl,2}\;\frac{\alpha_1^{(2)}\alpha_2^{(2)}}{2}\;(\kappa^2G^+)^2\,,
\end{aligned}
\tag{A8}
$$

respectively, where the superscript numbers represent the powers of (κH). To compute the correlation functions of the primordial perturbations in momentum space, it is convenient to use the Fourier-transformed Wightman function given by

$$
G^+_k(\tau_1,\;\tau_2)\equiv\psi_{f,\boldsymbol{k}}(\tau_1)\psi^*_{f,\boldsymbol{k}}(\tau_2)\,,
\tag{A9}
$$

where the mode function $\psi_{f,\boldsymbol{k}}(\tau)$ satisfies

$$
[\partial_0^2+2\mathcal{H}\partial_0+k^2+a^2V(\phi_{cl})\eta_V\kappa^2]\;\psi_{f,\boldsymbol{k}}(\tau)=0\,.
\tag{A10}
$$

1. Scalar perturbations

Using Eq. (A8), we give the leading loop corrections to the correlation function of the stochastic variable ξ_{ab}. To compute the loop corrections to the scalar perturbations, first we have to find the correlation function of the density perturbation of the stochastic variable, which is given by the $(0,0,0',0')$ component of the noise kernel. It is obtained from the real part of $F_{000'0'}$. The part of order of $(\kappa H)^4$ is

$$
\begin{aligned}
\hat F^{(4)0\;\;0'}{}_{0\;\;0'}(\tau_1,\;\tau_2,\boldsymbol{k},\boldsymbol{p}) \;\equiv\;& \int d^3\boldsymbol{x}_1\int d^3\boldsymbol{x}_2 e^{-i\boldsymbol{k}\cdot\boldsymbol{x}_1}e^{-i\boldsymbol{p}\cdot\boldsymbol{x}_2}F^{(4)0\;\;0'}{}_{0\;\;0'}(x_1,\;x_2)\\
=\;& \frac{1}{2}\,\delta^{(3)}(\boldsymbol{k}+\boldsymbol{p})\int d^3\boldsymbol{q}\Big\{(a_1)^{-2}\Big(\partial_{\tau_1}{}^q\partial_{\tau_1}{}^{k-q}-\boldsymbol{q}\cdot(\boldsymbol{k}-\boldsymbol{q})\Big)+\alpha_1^{(2)}V_{cl,1}\kappa^2\Big\}\\
&\times\Big\{(a_2)^{-2}\Big(\partial_{\tau_2}{}^q\partial_{\tau_2}{}^{k-q}-\boldsymbol{q}\cdot(\boldsymbol{k}-\boldsymbol{q})\Big)+\alpha_2^{(2)}V_{cl,2}\kappa^2\Big\}\;G^+_q(\tau_1,\;\tau_2)G^+_{|\boldsymbol{k}-\boldsymbol{q}|}(\tau_1,\;\tau_2)
\end{aligned}
\tag{A11}
$$

We put the momentum superscript on the partial derivative operator to represent that $\partial_{\tau_1}{}^q$ operates only to the Wightman function, $G^+_q(\tau_1,\;\tau_2)$. For example, $\partial_{\tau_1}{}^q\partial_{\tau_1}{}^{k-q}\;G^+_q(\tau_1,\;\tau_2)G^+_{|\boldsymbol{k}-\boldsymbol{q}|}(\tau_1,\;\tau_2)$ means $\partial_{\tau_1}\;G^+_q(\tau_1,\;\tau_2)\partial_{\tau_1}G^+_{|\boldsymbol{k}-\boldsymbol{q}|}(\tau_1,\;\tau_2)$. Taking into account that the correlation function of the stochastic variable is given by the noise kernel (2.2), we find Eq. (A11) implies

$$
\begin{aligned}
\langle\delta\rho_{\xi\boldsymbol{k}}(\tau_1)\delta\rho_{\xi\boldsymbol{p}}(\tau_2)\rangle^{(4)} \;=\;& \int d^3\boldsymbol{x}_1\int d^3\boldsymbol{x}_2e^{-i\boldsymbol{k}\cdot\boldsymbol{x}_1}e^{-i\boldsymbol{p}\cdot\boldsymbol{x}_2}\langle\xi^0{}_0(x_1)\xi^{0'}{}_{0'}(x_2)\rangle^{(4)}\\
=\;& \frac{1}{8}\Big[\hat F^{(4)0\;\;0'}{}_{0\;\;0'}(\tau_1,\;\tau_2,\boldsymbol{k},\boldsymbol{p})+\hat F^{(4)0\;\;0'}{}_{0\;\;0'}(\tau_1,\;\tau_2,\;-\boldsymbol{k},\;-\boldsymbol{p})^*\Big]\\
=\;& \frac{1}{8}\,\delta^{(3)}(\boldsymbol{k}+\boldsymbol{p})\int d^3\boldsymbol{q}\Big\{(a_1)^{-2}\Big(\partial_{\tau_1}{}^q\partial_{\tau_1}{}^{k-q}-\boldsymbol{q}\cdot(\boldsymbol{k}-\boldsymbol{q})\Big)+\alpha_1^{(2)}V_{cl,1}\kappa^2\Big\}\\
&\times\Big\{(a_2)^{-2}\Big(\partial_{\tau_2}{}^q\partial_{\tau_2}{}^{k-q}-\boldsymbol{q}\cdot(\boldsymbol{k}-\boldsymbol{q})\Big)+\alpha_2^{(2)}V_{cl,2}\kappa^2\Big\}\mathrm{Re}\Big[\;G^+_q(\tau_1,\;\tau_2)G^+_{|\boldsymbol{k}-\boldsymbol{q}|}(\tau_1,\;\tau_2)\Big]\,.
\end{aligned}
\tag{A12}
$$

On the third equality, we have transformed the momentum $\boldsymbol{q}$ to $-\boldsymbol{q}$ in the integral of $\hat{F}^{(4)0\ \ 0'}_{\ \ \ \ 0\ \ 0'}(\tau_1,\ \tau_2,\ -\boldsymbol{k},\ -\boldsymbol{p})^*$. This integral corresponds to the integral of the momentum of the internal line of the loop graph.

2. Tensor perturbations

Next we calculate the correlation function of the transverse traceless part of the anisotropic pressure for which we have to compute the loop corrections to the tensor perturbations. The correlation function of the pressure part of the stochastic variable is given by the bi-tensor $F^{(4)i\ \ l'}_{\ \ \ \ j\ \ m'}$ in momentum space as

$$\begin{aligned}\hat{F}^{(4)i\ \ l'}_{\ \ \ \ j\ \ m'}(\tau_1,\ \tau_2,\boldsymbol{k},\boldsymbol{p}) &= \int d^3\boldsymbol{x}_1 \int d^3\boldsymbol{x}_2 e^{-i\boldsymbol{k}\cdot\boldsymbol{x}_1} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}_2} F^{(4)i\ \ l'}_{\ \ \ \ j\ \ m'}(x_1,\ x_2) \\ &= \frac{1}{2}\,(a_1a_2)^{-2}\,\delta(\boldsymbol{k}+\boldsymbol{p})\int d^3\boldsymbol{q}\,\left[\{q^i(k_j-q_j)+q_j(k^i-q^i)\}\{q^{l'}(k_{m'}-q_{m'})+q_{m'}(k^{l'}-q^{l'})\}\right. \\ &\qquad \left.+\ (\text{trace part})\ \right]\ G^+_q(\tau_1,\ \tau_2)G^+_{|\boldsymbol{k}-\boldsymbol{q}|}(\tau_1,\ \tau_2)\,. \end{aligned} \tag{A13}$$

The transverse traceless part of any tensor on spatial flat hypersurface can be extracted by means of the projection operator, $P^i_{\ j}(\boldsymbol{k})$, as follows,

$$t_t{}^i{}_j(\boldsymbol{k}) = \left[P^i_{\ k}(\boldsymbol{k})P^l_{\ j}(\boldsymbol{k}) - \frac{1}{2}P^i_{\ j}(\boldsymbol{k})P^l_{\ k}(\boldsymbol{k})\right]\,t^k_{\ l}(\boldsymbol{k}) \qquad\qquad P^i_{\ j}(\boldsymbol{k}) \equiv \delta^i_{\ j} - \frac{k^ik_j}{k^2}\,. \tag{A14}$$

Operating the projection operator onto $\hat{F}^{(4)i\ \ l'}_{\ \ \ \ j\ \ m'}(\tau_1,\ \tau_2,\boldsymbol{k},\boldsymbol{p})$, we extract its tensor part as follows:

$$\begin{aligned}[\hat{F}^{(4)i\ \ l'}_{\ \ \ \ j\ \ m'}(\tau_1,\ \tau_2,\boldsymbol{k},\boldsymbol{p})]_{TT} &= \frac{1}{2(a_1a_2)^2}\,\delta(\boldsymbol{k}+\boldsymbol{p})\int d^3\boldsymbol{q}\ G^+_q(\tau_1,\ \tau_2)G^+_{|\boldsymbol{k}-\boldsymbol{q}|}(\tau_1,\ \tau_2) \\ &\quad\times\left\{-2q^iq_j + \delta^i_{\ j}\left(q^2 - \frac{(\boldsymbol{q}\cdot\boldsymbol{k})^2}{k^2}\right) - \frac{k^ik_j}{k^2}\left(q^2+\frac{(\boldsymbol{q}\cdot\boldsymbol{k})^2}{k^2}\right) + 2\frac{\boldsymbol{q}\cdot\boldsymbol{k}}{k^2}(k^iq_j+q^ik_j)\right\} \\ &\quad\times\left\{-2q^{l'}q_{m'} + \delta^{l'}_{\ m'}\left(q^2 - \frac{(\boldsymbol{q}\cdot\boldsymbol{k})^2}{k^2}\right) - \frac{k^{l'}k_{m'}}{k^2}\left(q^2+\frac{(\boldsymbol{q}\cdot\boldsymbol{k})^2}{k^2}\right) + 2\frac{\boldsymbol{q}\cdot\boldsymbol{k}}{k^2}(k^{l'}q_{m'}+q^{l'}k_{m'})\right\}\,. \end{aligned} \tag{A15}$$

Consequently, taking into account the definition of the noise kernel (2.2), we can give the correlation function of the transverse traceless part of the anisotropic pressure $p\tau^{(t)}_\xi$ by

$$\begin{aligned}&\langle p\pi^{(t)}_{\xi\boldsymbol{k}}(\tau_1)e^i_{\ j}(\boldsymbol{k})p\pi^{(t)}_{\xi\boldsymbol{p}}(\tau_2)e^{l'}_{\ m'}(\boldsymbol{p})\rangle^{(4)} \\ &\quad= \int d^3\boldsymbol{x}_1\int d^3\boldsymbol{x}_2 e^{-i\boldsymbol{k}\cdot\boldsymbol{x}_1}e^{-i\boldsymbol{p}\cdot\boldsymbol{x}_2}\ \langle\xi^i_{\ j}(x_1)\xi^{l'}_{\ m'}(x_2)\rangle^{(4)tt} \\ &\quad= \frac{1}{8}\,(a_1a_2)^{-2}\,\delta(\boldsymbol{k}+\boldsymbol{p})\int d^3\boldsymbol{q}\ Re\Big[\ G^+_q(\tau_1,\ \tau_2)G^+_{|\boldsymbol{k}-\boldsymbol{q}|}(\tau_1,\ \tau_2)\ \Big] \\ &\qquad\times\left\{-2q^iq_j + \delta^i_{\ j}\left(q^2 - \frac{(\boldsymbol{q}\cdot\boldsymbol{k})^2}{k^2}\right) - \frac{k^ik_j}{k^2}\left(q^2+\frac{(\boldsymbol{q}\cdot\boldsymbol{k})^2}{k^2}\right) + 2\frac{\boldsymbol{q}\cdot\boldsymbol{k}}{k^2}(k^iq_j+q^ik_j)\right\} \\ &\qquad\times\left\{-2q^{l'}q_{m'} + \delta^{l'}_{\ m'}\left(q^2 - \frac{(\boldsymbol{q}\cdot\boldsymbol{k})^2}{k^2}\right) - \frac{k^{l'}k_{m'}}{k^2}\left(q^2+\frac{(\boldsymbol{q}\cdot\boldsymbol{k})^2}{k^2}\right) + 2\frac{\boldsymbol{q}\cdot\boldsymbol{k}}{k^2}(k^{l'}q_{m'}+q^{l'}k_{m'})\right\}\,. \end{aligned} \tag{A16}$$

Especially when we contract the suffices $(i,\ m')$ and $(j,\ l')$, this correlation function is rewritten as

$$\begin{aligned}&\langle p\pi_{\xi\boldsymbol{k}}(\tau_1)e^i_{\ j}(\boldsymbol{k})\ p\pi_{\xi\boldsymbol{p}}(\tau_2)e^j_{\ i}(\boldsymbol{p})\rangle^{(4)} \\ &\quad= \frac{1}{4}\,(a_1a_2)^{-2}\,\delta(\boldsymbol{k}+\boldsymbol{p})\int d^3\boldsymbol{q}\ \left(q^2 - \frac{(\boldsymbol{k}\cdot\boldsymbol{q})^2}{k^2}\right)^2\ Re\Big[\ G^+_q(\tau_1,\ \tau_2)G^+_{|\boldsymbol{k}-\boldsymbol{q}|}(\tau_1,\ \tau_2)\ \Big]\,. \end{aligned} \tag{A17}$$

Appendix B: Loop integration

To integrate over the inner momentum, $\boldsymbol{q}$, it is convenient to consider the functions, $f(k,\delta,\mathcal{H})$ and $g(k,\delta,\mathcal{H})$ which are defined as

$$f(k,\delta,\mathcal{H}) \equiv \int_{q\in[k_i,\mathcal{H}]} d^3q \frac{1}{q^{3+\delta}} \frac{1}{|\boldsymbol{k}-\boldsymbol{q}|^{3+\delta}} \tag{B1}$$

$$g(k,\delta,\mathcal{H}) \equiv \int_{q\in[k_i,\mathcal{H}]} d^3q \Big\{q^2 - \frac{(\boldsymbol{k}\cdot\boldsymbol{q})^2}{k^2}\Big\}^2 \frac{1}{q^{3+\delta}} \frac{1}{|\boldsymbol{k}-\boldsymbol{q}|^{3+\delta}}\,, \tag{B2}$$

where δ is an arbitrary small constant. Here we assume that k_i is much smaller than k, i.e., $k_i \ll k$. In fact, k_i is defined by the horizon scale on the initial time as $k_i \equiv \mathcal{H}_i$. Since we are interested only in the modes whose scales are much smaller than the initial Hubble horizon scale, it is appropriate to assume $k_i \ll k$.
To make the integration simple, we approximate $|\boldsymbol{k}-\boldsymbol{q}|^{-(3+\delta)}$ as

$$\begin{aligned}
&\text{for } q<k \qquad \frac{1}{|\boldsymbol{k}-\boldsymbol{q}|^{3+\delta}} \simeq \frac{1}{k^{3+\delta}}\Big\{1+3\frac{\boldsymbol{k}\cdot\boldsymbol{q}}{k^2}+O\Big((q/k)^2\Big)\Big\}\\
&\text{for } k<q \qquad \frac{1}{|\boldsymbol{k}-\boldsymbol{q}|^{3+\delta}} \simeq \frac{1}{q^{3+\delta}}\Big\{1+3\frac{\boldsymbol{k}\cdot\boldsymbol{q}}{q^2}+O\Big((k/q)^2\Big)\Big\}\,.
\end{aligned} \tag{B3}$$

Then $f(k,\delta,\mathcal{H})$ is given by

$$\begin{aligned}
f(k,\delta,\mathcal{H}) &= 4\pi\Big[-\frac{1}{\delta}\frac{1}{k^{3+\delta}}\{(k-k_i)^{-\delta}-k_i^{-\delta}\} - \frac{1}{3+2\delta}\{\mathcal{H}^{-3-\delta}-(k+k_i)^{-3-2\delta}\}\Big]\\
&\simeq \frac{4\pi}{k^3}\Big[k^{-2\delta}\Big\{\frac{1}{3}-\frac{1}{\delta}\Big(1-\Big(\frac{k_i}{k}\Big)^{-\delta}\Big)\Big\} - \frac{1}{3}\Big(\frac{k}{\mathcal{H}}\Big)^3\mathcal{H}^{-2\delta}\Big]\,.
\end{aligned} \tag{B4}$$

Similarly, $g(k,\delta,\mathcal{H})$ is given by

$$\begin{aligned}
g(k,\delta,\mathcal{H}) &= \frac{32\pi}{15}\Big[\frac{1}{4-\delta}\frac{1}{k^{3+\delta}}\Big\{(k-k_i)^{4-\delta}-k_i^{4-\delta}\Big\} + \frac{1}{1-2\delta}\Big\{\mathcal{H}^{1-2\delta}-(k+k_i)^{1-2\delta}\Big\}\Big]\\
&\simeq \frac{32\pi}{15}\Big(\mathcal{H}^{1-2\delta}-\frac{3}{4}k^{1-2\delta}\Big)\,.
\end{aligned} \tag{B5}$$

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We have to note that if we adopt the different approach, we can show the equivalence in the linear perturbations between quantization in stochastic gravity and the conventional quantization by use of the gauge invariant variables [37, 38]. In this approach, when we quantize a scalar field, we have to quantize the gauge invariant variable which contains the longitudinal part of gravitational field. In this case, we do not encounter the above-mentioned problem even at superhorizon region, i.e. ζ gives the collect value. Although we obtain the same results in stochastic gravity as those in the conventional gauge invariant theory, there appears no stochastic gravitational field al least at the linear paerturbation level. The fluctuation of grvaitational field is included in the gauge invariant variable.
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